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## Electromagnetic form factors of the nucleon in a covariant diquark model

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### Abstract

We present a simple covariant constituent diquark-quark model for the nucleon. The nucleon is assumed to be composed of a scalar diquark and a quark which interact via a quark exchange. Starting from the Bethe-Salpeter equation, the instantaneous approximation leads to a diquark-quark Salpeter equation. In the Mandelstam formalism, the electromagnetic form factors of the nucleon are calculated for momentum transfers up to  $q^2 = -3 \text{ (GeV/c)}^2$ . A remarkable description of the experimental data is obtained. Especially, the model gives nearly the right values for the proton and (negative) neutron charge radii, and a qualitative description of the magnetic form factors.

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## I. INTRODUCTION

The possible existence of diquarks within the nucleon has been discussed for a long time. Undoubtly, there is much experimental evidence for diquarks or diquark correlations/clustering. For a review on that subject see e.g. [1–3]. One main argument for diquarks is the possible explanation of the negative neutron mean square charge radius [2], which cannot be explained in a pure SU(6) quark model.

On the other hand, of course, the analytical and numerical treatment of a two-body system is much less complicated than a three-body calculation. This is especially the case if one treats the problem relativistically using a Faddeev-type equation. In the last years, using the notion of diquarks, much progress has been done in this field. Especially, the description of baryons in the framework of the Nambu–Jona-Lasinio model [4–7] has been successful. There, two quarks within the baryon are treated as diquark states analogous to mesons (both are  $\bar{3}$  in colour space). The interaction with the third quark then is modelled by a quark exchange. Lichtenberg [8] proposed this short-range interaction long ago to split the degeneracy of the  $(70,0^+)$  and  $(56,0^+)$  ground state in a quark model of a SU(6) 21-plet diquark and a 6-plet quark. If one assumes a diquark clustering in the nucleon, it is very natural to assume such an interaction. In that case, none of the identical quarks is distinguished although we have diquark correlation. In [9–11] masses and diquark form factors are calculated within the NJL model. Cahill et al. [12] have calculated the masses of scalar and axial-vector diquarks by using an approximate form of the Bethe-Salpeter equation for qq bound states. Most of the above references conclude a probable dominant role of the scalar diquark compared to the axial-vector one. The parameters used in the above literature will serve us as a guideline.

Any relativistic model of the nucleon should of course reproduce correctly the electromagnetic form factors of the nucleon for non-zero momentum transfers. Meyer [13] solves numerically the BS equation for quark-diquark bound states with a quark exchange potential without, however, calculating any dynamical observables. Kroll et al. [14,15] successfully calculate form factors of the nucleon in a diquark model using light-cone distribution amplitudes. Of course, their model is applicable only at higher momentum transfers.

In our framework, form factors can be calculated in a straightforward way once the (covariant) amplitudes are known. We present a relativistic constituent quark model of the nucleon where the nucleon is assumed to be a bound state of a scalar diquark and a quark. Following the motivation given above, the interaction between the two particles is modelled by the exchange of a quark. Since we are only interested in the (ground state) nucleons, we do not incorporate a phenomenological confinement potential. We just want to show, that, by using only a (short-range) quark exchange potential, it is possible to describe the form factors with physically acceptable parameters. The fundamental equation is the Bethe-Salpeter equation for a bound state of a scalar particle and a fermion. Assuming an instantaneous interaction kernel we derive a Salpeter-type equation. This can be cast in a Schrödinger-type

equation. This eigenvalue equation is solved numerically by standard techniques. The formal covariance of the Salpeter equation and the Salpeter amplitudes is discussed extensively in [16]. We then use a straightforward method to calculate the electromagnetic currents in the Mandelstam formalism, analogous to [16] where meson form factors are computed. Since form factors are directly related to wave functions, the calculation of them is a crucial test for any model.

This paper is structured as follows. In Sect.II we derive the Salpeter equation for a bound state of a scalar diquark and a quark, including the normalization of the amplitudes. The assumed interaction between diquark and quark is outlined in Sect.II B. In Sect.II C the procedure solving the Salpeter equation is explained. The computation of the electromagnetic current and of the resulting form factors of the nucleon is discussed in Sect.III. The results for the form factors are presented in Sect.IV and compared to experimental data. Finally, in Sect.V, a summary is given, and an outlook to an extension of the model and to possible applications.

## II. THE MODEL

### A. The Bethe-Salpeter equation

A relativistic bound state of a scalar particle and a fermion with four momentum  $P$  is described by a Bethe-Salpeter amplitude

$$\chi_P(x_1, x_2) = \langle 0 | T\phi(x_1)\psi(x_2) | P \rangle \quad . \quad (1)$$

In momentum space,  $\chi_P$  fulfills the following Bethe-Salpeter equation [17]

$$\chi_P(p) = \Delta_1^F(p_1)S_2^F(p_2) \int \frac{d^4p'}{(2\pi)^4} (-iK(P, p, p'))\chi_P(p') \quad (2)$$

with  $\Delta_1^F(p_1) = i/(p_1^2 - m_1 + i\epsilon)$  and  $S_2^F(p_2) = i/(\gamma p_2 - m_2 + i\epsilon)$  the usual Feynman propagators for a scalar particle and a quark, respectively, and  $-iK$  the irreducible interaction kernel. The masses in the above propagators are constituent masses of the order of a few hundred MeV. For the momenta we set

$$p_1 = \eta_1 P + p, \quad p_2 = \eta_2 P - p \quad ; \quad P = p_1 + p_2, \quad p = \eta_2 p_1 - \eta_1 p_2 \\ \text{with } \eta_1 + \eta_2 = 1 \quad . \quad (3)$$

The normalization condition is obtained as usual by considering the pole contribution of the two-particle (4-point-) Green's function (see e.g. [18])

$$G(x_1, x_2, x'_1, x'_2) = \langle 0 | T\phi(x_1)\psi(x_2)\phi^\dagger(x'_1)\bar{\psi}(x'_2) | 0 \rangle \quad (4)$$

to the bound state:

$$G(P, p, p') \sim \frac{i}{(2\pi)^4} \frac{\chi_P(p) \cdot \bar{\chi}_P(p')}{2\omega_P(P^0 - \omega_P + i\epsilon)} \quad . \quad (5)$$

$\omega_P = \sqrt{\vec{P}^2 + M^2}$  is the on-shell energy of the bound state. This yields

$$\frac{i}{(2\pi)^4} \int d^4p \int \frac{d^4p'}{(2\pi)^4} \bar{\chi}_P(p) \frac{\partial}{\partial P^0} [H(P, p, p')]_{P^0=\omega_P} \chi_P(p') = 2\omega_P \quad (6)$$

with

$$H(P, p, p') = \Delta_1^F(p_1)^{-1} S_2^F(p_2)^{-1} (2\pi)^4 \delta^4(p - p') + iK(P, p, p') \quad . \quad (7)$$

In covariant form, Eq.6 reads:

$$\frac{i}{(2\pi)^4} \int d^4p \int \frac{d^4p'}{(2\pi)^4} \bar{\chi}_P(p) P^\mu \frac{\partial}{\partial P^\mu} [H(P, p, p')]_{P^0=\omega_P} \chi_P(p') = 2M^2 \quad (8)$$

Following Salpeter [19] we neglect the time (i.e. energy) dependence of the interaction kernel by assuming  $K(P, p, p') = V(\vec{p}, \vec{p}')$  (*instantaneous interaction*) in the rest frame of the bound state. Then, one can easily perform the  $p^0$  integration: Define in the c.m. frame of the bound state

$$\Phi(\vec{p}) = \left( \int \frac{dp^0}{2\pi} \chi_P(p^0, \vec{p}) \right)_{P=(M, \vec{0})} \quad . \quad (9)$$

Then, from Eq.2 one gets with standard techniques

$$\Phi(\vec{p}) = \frac{1}{2\omega_1} \left( \frac{\Lambda_2^+(-\vec{p})\gamma^0}{M - \omega_1 - \omega_2} + \frac{\Lambda_2^-(-\vec{p})\gamma^0}{M + \omega_1 + \omega_2} \right) \int \frac{d^3p'}{(2\pi)^3} V(\vec{p}, \vec{p}') \Phi(\vec{p}') \quad , \quad (10)$$

with  $\Lambda^\pm(\vec{p}) = \frac{\omega \pm H(\vec{p})}{2\omega}$  the usual Dirac projectors.  $H(\vec{p}) = \gamma^0(\vec{\gamma}\vec{p} + m)$  is the Dirac Hamiltonian. Define now

$$\begin{aligned} \Psi(\vec{p}) &= \gamma^0 \Phi(\vec{p}) \\ W(\vec{p}, \vec{p}') &= V(\vec{p}, \vec{p}') \gamma^0 \quad . \end{aligned} \quad (11)$$

Then, we can rewrite Eq.10 in a Schrödinger-type equation:

$$\begin{aligned} (\mathcal{H}\Psi)(\vec{p}) &= M\Psi(\vec{p}) \\ &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi(\vec{p}) + \frac{1}{2\omega_1} \int \frac{d^3p'}{(2\pi)^3} W(\vec{p}, \vec{p}') \Psi(\vec{p}') \\ &= (\mathcal{T} + \mathcal{V}) \Psi(\vec{p}) \quad . \end{aligned} \quad (12)$$

This is the fundamental equation of our model.  $H_2(\vec{p})$  is again the Dirac-Hamiltonian of the quark. If  $\Psi$  were a free Dirac spinor,  $\Psi(\vec{p}) = u(\vec{p})$ , the kinetic energy term in Eq.12 is just the sum of the kinetic energies of the two particles:  $\mathcal{T} = \omega_1 + \omega_2$ . In a similar way, one obtains the normalization condition for the instantaneous approximation from Eq.8:

$$\frac{i}{(2\pi)^4} \int d^4p \bar{\chi}_P(p) P^\mu \frac{\partial}{\partial P^\mu} (\Delta_1^F(p_1)^{-1} S_2^F(p_2)^{-1}) \chi_P(p') = 2M^2 \quad . \quad (13)$$

Performing the  $p^0$  integration, one ends up with a familiar normalization of the functions  $\Phi(\vec{p})$ :

$$\int \frac{d^3p}{(2\pi)^3} (2\omega_1) \bar{\Phi}(\vec{p}) \gamma^0 \Phi(\vec{p}) = 2M \quad . \quad (14)$$

It is easy to see that  $\bar{\Phi}(\vec{p}) = \Phi^\dagger(\vec{p}) \gamma^0$ . So apart from the factor  $2\omega_1$ , we obtain the usual normalization of Dirac spinors. In particular, the functions  $\Psi$  solving Eq.12 have non-negative norm for  $M > 0$ . We thus define a *scalar product* as follows (compare [20] in the case of two fermions):

$$\langle \phi | \psi \rangle = \int \frac{d^3p}{(2\pi)^3} (2\omega_1) \bar{\phi}(\vec{p}) \gamma^0 \psi(\vec{p}) \quad . \quad (15)$$

In order to obtain a pseudo-Hamiltonian  $\mathcal{H}$  which is hermitian with respect to the above scalar product, the kernel  $W(\vec{p}, \vec{p}')$  has to fulfill the following condition:

$$\begin{aligned} W(\vec{p}, \vec{p}')^\dagger &= W(\vec{p}', \vec{p}) \\ \text{or } V(\vec{p}, \vec{p}')^\dagger &= \gamma^0 V(\vec{p}', \vec{p}) \gamma^0 \quad . \end{aligned} \quad (16)$$

In general, taking into account also anti-particle states, the solutions of Eq.12 come in pairs. This can be seen as follows. Under charge conjugation, the (Bethe-)Salpeter amplitudes transform like (see Eq.A5)

$$\begin{aligned} \chi_P^{b_1 f_2}(p) &= S_C \bar{\chi}_P^{\bar{b}_1 \bar{f}_2}(-p)^* \\ \rightarrow \Psi_{b_1 f_2}(\vec{p}) &= -S_C \Psi_{\bar{b}_1 \bar{f}_2}^*(-\vec{p}) \end{aligned} \quad (17)$$

with  $S_C = i\gamma^2$ . With  $b_1, f_2$  and  $\bar{b}_1, \bar{f}_2$  we denote the quantum numbers of the diquark and quark and their antiparticles, respectively. With  $S_C H_2(\vec{p}) S_C = -^t H_2(-\vec{p})$  and the imposed condition

$$W(\vec{p}, \vec{p}') = -S_C^t W(-\vec{p}, -\vec{p}') S_C \quad (18)$$

Eq.12 results in

$$\begin{aligned} (\mathcal{H} \Psi_{\bar{b}_1 \bar{f}_2})(\vec{p}) &= -M^* \Psi_{\bar{b}_1 \bar{f}_2}(\vec{p}) \\ &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi_{\bar{b}_1 \bar{f}_2}(\vec{p}) + \frac{1}{2\omega_1} \int \frac{d^3p'}{(2\pi)^3} W(\vec{p}, \vec{p}') \Psi_{\bar{b}_1 \bar{f}_2}(\vec{p}') \quad . \end{aligned} \quad (19)$$

## B. The interaction kernel

The interaction between the scalar diquark and the quark is assumed to be a quark exchange, see Fig. 1. The diquark couples pointlike to the two quarks according to the Lagrangian [13]

$$\mathcal{L}_{int} = -i g_s \bar{\psi}_c \gamma^5 \tau_2 \psi \phi^* \quad . \quad (20)$$

The kernel  $K(P, p, p')$  then is

$$\begin{aligned} K(P, p, p') \chi_P(p') &= -i g_{dqq}^2 S_q^F(p + p') \chi_P(p') \\ &\rightarrow V(\vec{p}, \vec{p}') = -g_{dqq}^2 \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \quad , \end{aligned} \quad (21)$$

with  $\omega_q = \sqrt{(\vec{p} + \vec{p}')^2 + m_q^2}$  the energy of the exchanged quark; set  $m_q = m_2$  (mass of the quark). In the instantaneous approximation we neglect the  $p^0$  dependence of the quark propagator.  $g_{dqq}$  is a dimensionless diquark-quark coupling constant. Obviously,  $V(\vec{p}, \vec{p}')$  fulfills the relation 16. In addition, it does not mix states with different parities since

$$V(\vec{p}, \vec{p}') = \gamma^0 V(-\vec{p}, -\vec{p}') \gamma^0 \quad (22)$$

and Eq.18 holds for the above  $W(\vec{p}, \vec{p}') = V(\vec{p}, \vec{p}') \gamma^0$ .

## C. Solving the instantaneous diquark-quark equation

Since nucleons have positive parity and spin  $\frac{1}{2}$  we choose as basis in Dirac space the following states ( $s = \pm\frac{1}{2}$ ):

$$\begin{aligned} e_s^1(\vec{p}) &= \sqrt{W + M_{12}} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} \\ e_s^2(\vec{p}) &= \sqrt{W + M_{12}} \begin{pmatrix} 0 \\ \frac{\vec{\sigma}\vec{p}}{W + M_{12}} \chi_s \end{pmatrix} \quad , \end{aligned} \quad (23)$$

with  $M_{12} = \frac{m_1 m_2}{m_1 + m_2}$  and  $W = \sqrt{\vec{p}^2 + M_{12}^2}$ . This ansatz neglects a relative angular momentum between the quark and the diquark. Both states are multiplied by independent radial functions which are chosen to be eigenstates of the harmonic oscillator:  $\mathcal{R}^i(p) = (-1)^i \tilde{N}_{i0} \sum_{\mu=0}^i c_{\mu}^{i0} p^{2\mu} \exp(-\frac{1}{2}\alpha^2 p^2)$ .  $\alpha$  is the oscillator parameter. Eq.12 is then diagonalized. The lowest positive eigenvalue is interpreted as the nucleon mass. It is obtained according to the Ritz variational principle by choosing the oscillator parameter  $\alpha$  in the minimum of the curve  $M(\alpha)$ , see Fig.3. The coupling  $g_{dqq}$  is determined by the condition

$M = 939 \text{ MeV}/c^2$ . We observe that for a pointlike coupling of the two quarks to the diquark, there is no stable solution. Only if one introduces a diquark form factor the minimum of the  $M(\alpha)$  curve converges with increasing basis states, see Fig.3. We choose a form factor of the following form:

$$F(\vec{q}) = \exp(-\lambda^2 \vec{q}^2) \quad (24)$$

and put  $\lambda = 0.18 \text{ fm}$ , see Sect.IV, where we study the dependence of our results on  $\lambda$ .

### III. ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON

In this section we calculate the electromagnetic form factors of the nucleon in the Mandelstam formalism [21,22]. In this framework, any current matrix elements between bound states can be calculated in a covariant way [16]. In our model, the electromagnetic current coupling to the nucleon is simply the sum of the diquark- and quark current, see Fig.2. For the coupling to the quark one obtains (put  $\eta_1 = \eta_2 = \frac{1}{2}$ )

$$\langle P' s' | j_\mu^{quark}(x=0) | P s \rangle = e_q \int \frac{d^4 p}{(2\pi)^4} \bar{\chi}_{P'}(p + \frac{q}{2}) \gamma_\mu \Delta_1^F(\frac{P}{2} + p)^{-1} \chi_P(p) \quad . \quad (25)$$

Now introduce the vertex functions as amputated Salpeter amplitudes:

$$\begin{aligned} \Gamma_P(p) &= \Delta_1^F(p_1)^{-1} S_2^F(p_2)^{-1} \chi_P(p) \\ \bar{\Gamma}_P(p) &= \bar{\chi}_P(p) \Delta_1^F(p_1)^{-1} S_2^F(p_2)^{-1} \quad . \end{aligned} \quad (26)$$

From Eq.2 follows in the instantaneous approximation

$$\Gamma(\vec{p}) := \Gamma_P(p)|_{P=(M,\vec{0})} = -i \int \frac{d^3 p'}{(2\pi)^3} V(\vec{p}, \vec{p}') \Phi(\vec{p}') \quad (27)$$

$$\text{and } \bar{\Gamma}(\vec{p}) = -\Gamma^\dagger(\vec{p}) \gamma^0 \quad . \quad (28)$$

Then, the rhs. of Eq.25 becomes

$$= e_q \int \frac{d^4 p}{(2\pi)^4} \bar{\Gamma}_{P'}(p + \frac{q}{2}) S_2^F(\frac{P}{2} - p - q) \gamma_\mu S_2^F(\frac{P}{2} - p) \Gamma_P(p) \Delta_1^F(\frac{P}{2} + p) \quad . \quad (29)$$

Analogously, for the current coupling to the scalar diquark, one obtains

$$\begin{aligned} \langle P' s' | j_\mu^{diquark}(0) | P s \rangle &= e_d \int \frac{d^4 p}{(2\pi)^4} \bar{\chi}_{P'}(p - \frac{q}{2}) S_2^F(\frac{P}{2} - p)^{-1} (p_1 + p'_1)_\mu \chi_P(p) \\ &= e_d \int \frac{d^4 p}{(2\pi)^4} \bar{\Gamma}_{P'}(p - \frac{q}{2}) S_2^F(\frac{P}{2} - p) \Gamma_P(p) \Delta_1^F(\frac{P}{2} + p) (P + 2p - q)_\mu \Delta_1^F(\frac{P}{2} + p - q) \quad , \end{aligned} \quad (30)$$

where we assumed a coupling of the photon to a pointlike scalar particle. Since the vertex functions  $\Gamma(\vec{p})$  are only known in the nucleon rest frame (Eq.27), the vertex function of the outgoing nucleon has to be boosted according to the value of  $q^2$ , see Eq.A1. This covariant treatment of the vertex functions is very important for high  $q^2$ . The  $p^0$  integration in Eqs.29,30 has the generic form

$$\int_{-\infty}^{+\infty} dp^0 \frac{f(p^0)}{\prod_{i=1,6} (p^0 - p_i^0 \pm i\epsilon)} \quad (31)$$

with  $f(p^0)$  a regular function. The six poles arise from the denominators of the three propagators. The integral can be split in a residue part and a principal value integral. To obtain a hermitian current the latter should vanish. The  $\phi_p$  integration is trivial, since  $p$  and  $p' = p \pm \frac{q}{2}$  have the same  $\phi_p$ . The numerical treatment is further simplified by expanding the vertex functions in the basis of Eq.23:

$$\Gamma_s(\vec{p}) = \sum_i a_i^1(p) e_s^1 + a_i^2(p) e_s^2(\vec{p}) \quad , \quad (32)$$

where  $\sum_i$  sums over the radial basis states.

We recall that the normalization condition (Eq.14) equals the zero-component of the currents at  $q^2 = 0$ :

$$\begin{aligned} \frac{1}{e_q} \langle P s | j_0^{quark} | P s \rangle &= \frac{1}{e_d} \langle P s | j_0^{diquark} | P s \rangle = 2M \\ &= - \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_1} \left( \frac{\bar{\Gamma}(\vec{p}) \Lambda_2^+(-\vec{p}) \gamma^0 \Gamma(\vec{p})}{(M - \omega_1 - \omega_2)^2} + \frac{\bar{\Gamma}(\vec{p}) \Lambda_2^-(-\vec{p}) \gamma^0 \Gamma(\vec{p})}{(M + \omega_1 + \omega_2)^2} \right). \end{aligned} \quad (33)$$

The nucleon electromagnetic current can be written in the usual form [23]

$$\langle P' s' | j_\mu(0) | P s \rangle = e \bar{u}_{s'}(P') \left[ \gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} \kappa_N F_2^N(q^2) \right] u_s(P) \quad , \quad (34)$$

with  $F_1^N$  and  $F_2^N$  being independent form factors. At  $q^2 = 0$  we have

$$F_1^p(0) = \frac{1}{e} \frac{\langle P s | j_0^{quark} | P s \rangle + \langle P s | j_0^{diquark} | P s \rangle}{2M} = 1 \quad , \quad (35)$$

which thus serves as a check of the numerical evaluation of the currents at zero momentum transfer, i.e. of the normalization. Usually, one defines the following form factors:

$$\begin{aligned} G_E^N(q^2) &= F_1^N(q^2) + \frac{\kappa_N q^2}{4M^2} F_2^N(q^2) \\ G_M^N(q^2) &= F_1^N(q^2) + \kappa_N F_2^N(q^2) \quad . \end{aligned} \quad (36)$$

The direction of the (outgoing) photon momentum is chosen parallel to the z-axis, and the initial nucleon is put in its rest frame. Then,  $G_E^N$  and  $G_M^N$  can be calculated by solving the

set of equations for  $j_0$  and  $j_+ = \frac{1}{2}(j_1 + ij_2)$ . Of course, the continuity equation has to be fulfilled, which serves as a check of the calculation:

$$\partial^\mu j_\mu = \partial^0 j_0 + \partial^3 j_3 = 0 \quad (37)$$

In our framework the continuity equation follows directly from  $(\mu = 0, 3)$

$$\begin{aligned} \langle P' s' | j_\mu | P s \rangle &= (-1)^{1-s-s'} \langle P - s | j_\mu | P' - s' \rangle \\ &= (-1)^{1-s-s'} \langle P s | j_\mu | P' s' \rangle \quad , \end{aligned} \quad (38)$$

where the transformation properties of Eqs.29,30 under time reversal and parity transformation (App.A) have been used. Indeed, we find that the current is conserved numerically for any momentum transfers.

#### IV. RESULTS AND DISCUSSION

The parameter values used in our model are given in Tab.I. The parameter influencing the shape of the form factors mostly is the diquark parameter  $\lambda$  (Eq.24). After determining the other parameters,  $\lambda$  is chosen to give a best quantitative description of the proton electric form factor. The obtained value is in agreement with the commonly used diquark size in the range  $0.1 - 0.3$  fm [1,15,3]. The quark and diquark masses are free parameters: the constituent quark mass is chosen to be in a range adopted by most quark models, see e.g. [24]. The spectator diquark mass is a little less than the sum of two quarks [1,9,4]. These mass values agree with a binding energy of the nucleon proposed by the work of Ishii et al. [4] of 50 to 150 MeV. Thus, the nucleon appears as a loosely bound state of quarks. As outlined in Sect.II C, the oscillator parameter is given by the minimum of the  $M(\alpha)$  curve, and the coupling  $g_{dq}$  is adjusted by fitting the minimum to the nucleon mass of 939 MeV. Note that the so obtained oscillator parameter  $\alpha = 1.15$  fm coincides roughly with the assumed radius of the nucleon of about 1 fm [24]. Our calculation contains 6 radial basis functions, which is a sufficient approximation to the solution of Eq.12. The convergence of the solution is depicted in Fig.3.

The calculated electric form factor of the proton  $G_E^p$  as a function of the negative photon momentum squared is shown in Fig.4. For  $\lambda = 0.18$  fm we find a very good agreement with the experimental data (taken from [25]). Even for  $q^2$  as high as  $-3$  GeV<sup>2</sup> the calculation fits the data well. From the slope of  $G_E^p(q^2)$  at  $q^2 = 0$  we obtain for the square root of the mean square charge radius

$$\sqrt{\langle r^2 \rangle_p} = \left( -6 \frac{dG_E^p(q^2)}{dq^2} \Big|_{q^2=0} \right)^{\frac{1}{2}} = 0.88 \text{ fm} \quad , \quad (39)$$

compared to the experimental value  $\sqrt{\langle r^2 \rangle_p} = (0.862 \pm 0.012)$  fm [26]. The electric neutron form factor is compared to experiment [27,28] in Fig.5. We obtain a rather good description of the data, with a negative mean square charge radius

$$\langle r^2 \rangle_n = -0.28 \text{ fm}^2 \quad , \quad (40)$$

which is slightly larger than the experimental  $\langle r^2 \rangle_n = (-0.117 \pm 0.002) \text{ fm}^2$  [2]. The magnetic form factors of the proton and the neutron are shown in Fig.6. Our results are a factor of about 2 too small. However, the calculation matches the experimental data qualitatively. The shape of the curves is correct, and we obtain the right signs. Of course, since  $G_M$  is connected with the spin current, effects of the vector component of a two-quark subsystem should be important. Having neglected it, it is even more surprising to find such a good correspondence. Looking at  $G_M(0)$ , we obtain for the ratio of the magnetic moments of neutron and proton

$$\frac{\mu_n}{\mu_p} = -0.5 \quad , \quad (41)$$

compared to the experimental value  $-0.685$ . Of course, the above ratio arises from the different charges of the single quark in the nucleons; the scalar diquark does not contribute to the magnetic form factor. The prediction of the right ratio  $\frac{\mu_n}{\mu_p}$  is one of the successes of the SU(6) quark model. Thus, it would be interesting to study the effects of an additional axial-vector diquark component. It should be noted that the neutron form factors are related to the differences between the currents

$$j_\mu^n \sim \frac{1}{3} j_\mu^{diquark} - \frac{1}{3} j_\mu^{quark} \quad . \quad (42)$$

Thus, a description of the neutron form factors is most sensible to the wave function.

In Fig.7 we present the electric form factor of the proton for three different diquark extensions  $\lambda$ . The qualitative structure of the curve remains. Fig.8 shows the variation of the electric form factor for three different values of the diquark mass, with constant  $\lambda = 0.18$  fm. Obviously, the variation is small. However, we find that the diquark mass should not be smaller than about 620 MeV: for  $m_d = 600$  MeV the oscillator parameter is  $\alpha = 1.5$  fm, and rises sharply with decreasing  $m_d$ .

## V. SUMMARY AND OUTLOOK

We developed a simple, fully relativistic quark-diquark model of the nucleon. In a first step, only scalar diquarks are taken into account. Neglecting retardation effects of the interaction kernel, a Salpeter equation for a bound state of a constituent quark and a scalar particle is obtained. We assume a quark exchange as the only interaction. The electric

and magnetic form factors of the nucleon are then calculated in a straightforward way using the Mandelstam formalism. With four parameters (constituent quark and diquark masses, diquark extension  $\lambda$ , diquark-quark coupling  $g_{dqq}$ , see Tab.I), we obtain a very good description of the electric form factor up to  $q^2 = -3 \text{ (GeV/c)}^2$ . The neutron electric form factor is reproduced almost quantitatively. The magnetic form factors are in qualitative agreement with experiment.

Obviously, a relativistic treatment is extremely important to describe observables of the nucleon in the medium energy range. Furthermore, the assumption of a diquark structure of the nucleon seems to be not out of place.

Of course, it would be interesting to study the effects of eventual axial-vector diquarks. Especially for the magnetic form factors they should be important. - The model will be extended to baryons with strangeness. Then, various electromagnetic processes may be studied. An example is the photoproduction of kaons. The contributing amplitudes can be calculated in the framework of the Mandelstam formalism, and a direct coupling of the photon to the internal quark- and diquark lines is assumed (see [30]).

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# TABLES

$m_q$	$m_d$	$g_{dqq}$	$\lambda$
350 MeV/c <sup>2</sup>	650 MeV/c <sup>2</sup>	14.15	1.8 fm

TABLE I. The parameters of the model

# FIGURES

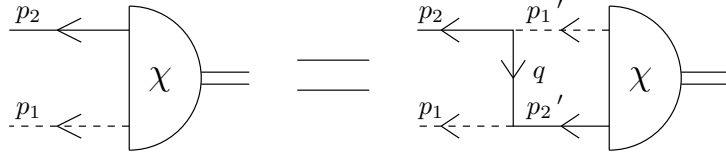


FIG. 1. The Bethe-Salpeter equation for a bound state of a quark and a scalar diquark with a quark exchange interaction

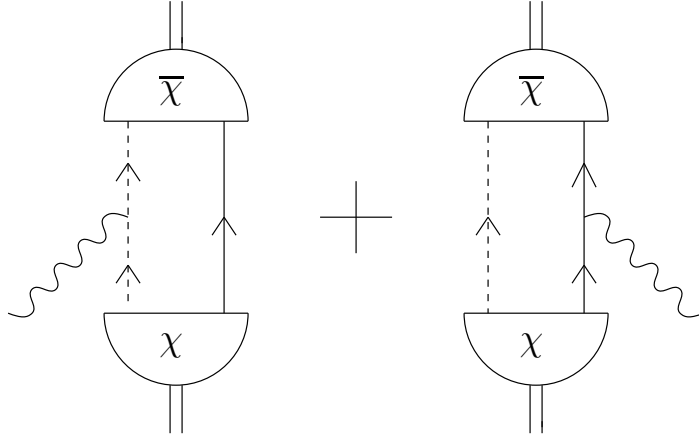


FIG. 2. The electromagnetic current is the sum of the diquark current and the quark current (Eqs. 25,30)

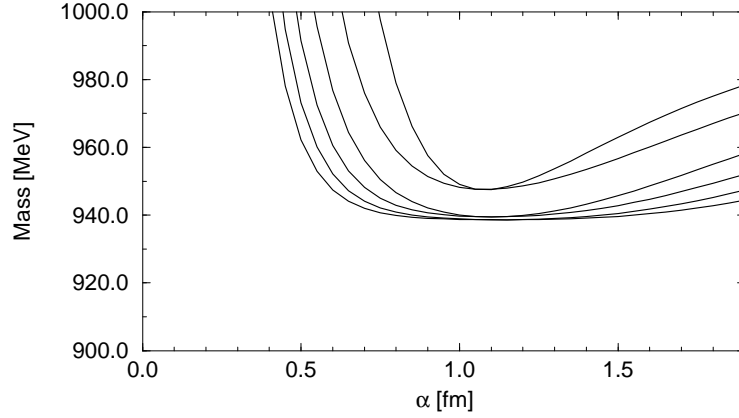


FIG. 3. The lowest positive eigenvalue of  $\mathcal{H}$  (Eq.12) as a function of the oscillator parameter  $\alpha$  for different numbers of basis functions (1 to 6 from top to bottom)

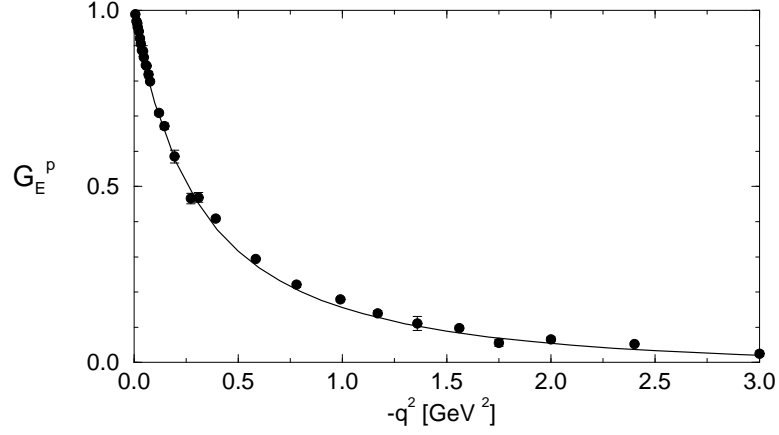


FIG. 4. The electric form factor of the proton  $G_E^p(q^2)$ ; experimental data are taken from [25]

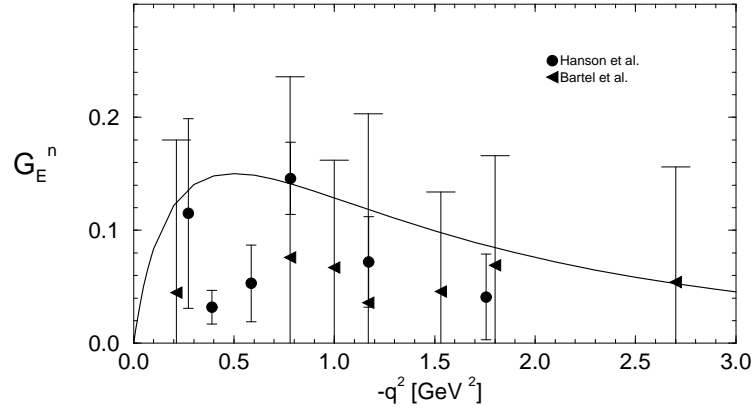


FIG. 5. The electric form factor of the neutron  $G_E^n(q^2)$ ; experimental data are taken from [27,28]

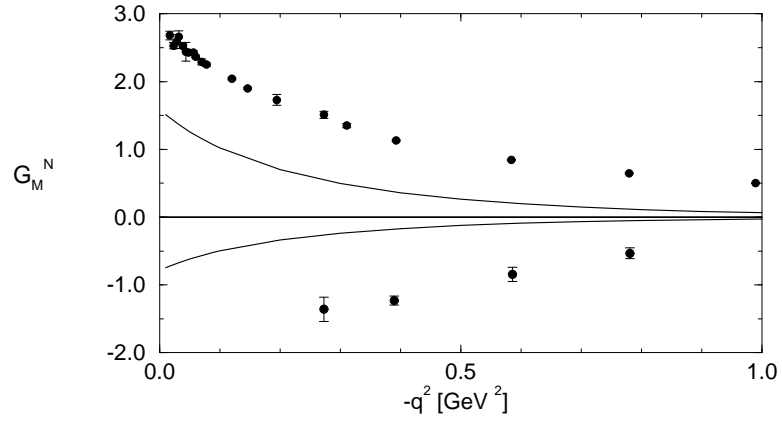


FIG. 6. The magnetic form factors  $G_M^N(q^2)$  of the proton (upper curve) and of the neutron (lower curve); experimental data are taken from [25,27]

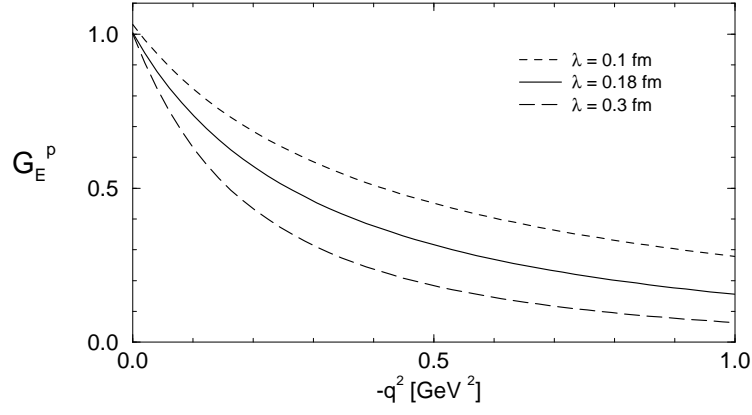


FIG. 7. The electric form factor of the proton  $G_E^p(q^2)$  for three different values of the diquark parameter  $\lambda$

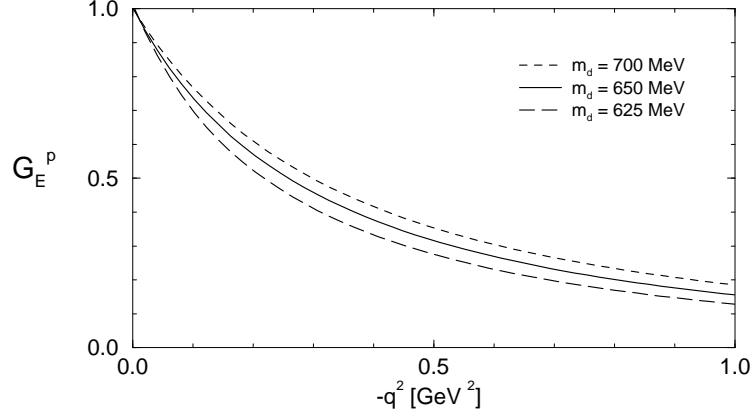


FIG. 8. The electric form factor of the proton  $G_E^p(q^2)$  for three different values of the diquark mass

## APPENDIX A: TRANSFORMATION PROPERTIES

Under a Lorentz transformation  $\Lambda$ , the Bethe-Salpeter amplitude for a bound state of a scalar particle and a fermion transforms like a Dirac spinor

$$\chi_P(p) = S_\Lambda^{-1} \chi_{\Lambda P}(\Lambda p) \quad . \quad (\text{A1})$$

Using  $S^F(\Lambda p) = S_\Lambda S^F(p) S_\Lambda^{-1}$  we obtain

$$\begin{aligned} \Gamma_P(p) &= S_\Lambda^{-1} \Gamma_{\Lambda P}(\Lambda p) \\ \bar{\Gamma}_P(p) &= \bar{\Gamma}_{\Lambda P}(\Lambda p) S_\Lambda \end{aligned} \quad (\text{A2})$$

Under time reversal (using the standard convention like e.g. in [29]) we find

$$\chi_P^s(p) = (-1)^{S-s} (-\epsilon) {}^t \bar{\chi}_{\tilde{P}}^{-s}(\tilde{p}) \quad (\text{A3})$$

with  $S = \frac{1}{2}$  for the nucleon and  $\epsilon = i\gamma^1\gamma^3\gamma^0$ .

Under parity transformation we have

$$\chi_P(p) = \pi_P \gamma^0 \chi_{\tilde{P}}(\tilde{p}) \quad , \quad (\text{A4})$$

and under charge conjugation:

$$\chi_P^{b_1 f_2}(p) = S_C \chi_P^{\bar{b}_1 \bar{f}_2}(-p)^* \quad (\text{A5})$$

with  $S_C = i\gamma^2$ .

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